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Open space preservation in an urbanization context

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Abstract

The objective of this paper is to address the question of open space preservation in an urbanization context. We study the possibility of preserving two different types of open spaces, large open spaces at cities' outskirt and small intra-urban open spaces. Thus we contribute to the debate of land sharing versus land sparing in a urban context. We analyze these questions by way of a theoretical microeconomics framework taking into account both households' preferences for open space and regulator's interest for the preservation of natural habitat for biodiversity. We compare land use patterns at private equilibrium and when the social planner maximizes social welfare.

JEL classification : Q24, R14, R28
Keywords : Urbanization, Open space preservation, Public intervention

1 Introduction

Since the publication of the report entitled "Urban sprawl : the ignored challenge" by the European Commission which concludes that urban sprawl is responsible for a lot of environmental degradation in urban areas, fighting urban sprawl has become an important objective of institutions and city planners (European Environment Agency, 2006). Creating compact cities is often recommended by planners, notably in order to avoid disturbance of natural open spaces at cities' outskirt. However, in this fight against urban sprawl, the importance of green space inside the urban areas has been put aside. Nonetheless, open space located inside cities' boundaries represents a significant share of land and deserve to be taken into account in the analysis of environmental preservation and the search of an optimal city structure. Indeed, in Stokholm for example, given the European data from the Urban Audit, green urban areas appear to represent $24,1\%$ of land. Green urban areas include public green areas, predominant recreational use as gardens, zoos, parks, castle parks, and suburban natural areas that have become and are managed as urban parks. If we add natural land that are not managed as urban parks, the total share of intra urban open spaces rise to more than 35%. The proportion of intra urban open spaces is quite similar for the city of Berlin, is also very important for Paris (almost 25%) but is less for others cities like Amsterdam $(14,5\%)$(Eurostat, 2017). In France, public authorities have recognized the role of these open spaces, as illustrated by the circular dating from February 8th 1973 which fixed precise objectives regarding the accessibility of open spaces : each urban dweller should have access to $10m^2$ of "proximity open spaces" located inside the cities such as defined in the green urban spaces of urban audit, and $25m^2$ of "weekend open spaces" such as forests or natural land located farther from their home.

We see that a possible contradiction may arise between the willingness to fight urban sprawl in order to preserve large open spaces at city's outskirt, synonymous with the promotion of a compact city, and the preservation of intra-urban open spaces in small patches that could be responsible for an extension of the city's limit. This question fits in the recent land sharing versus land sparing framework. This framework was first proposed by Green et al. (2005) in the literature on the impact of agriculture on biodiversity and aimed at understanding the extent to which agriculture should be concentrated on intensively farmed land in order to conserve more biodiversity-rich natural spaces elsewhere (*land sparing*) or conversely, be more wildlife-friendly but less productive, hence conserving fewer natural spaces (*land sharing*). This framework was extended to the context of cities recently in the ecological literature: land sparing minimizes the spatial extent of developed areas, such that residential areas are developed as intensively as possible, enabling the maintenance of large open spaces. Under land sharing, development is more evenly but less intensively distributed, such that a larger land area is needed to accommodate a given number of households, and open spaces tend to be more fragmented but on average closer to residential areas (Brenda and Fuller, 2013; Soga et al., 2014; Stott et al., 2015). From an ecological perspective, the question of which urban structure is preferable for biodiversity conservation is far from being completely answered. To broaden the debate, we propose in this paper an economic analysis of these questions, in order to understand how the behavior of economic agents influences the city's structure and the existence and preservation of different types of open space.

Several papers have already studied the effect of open spaces in a spatial urban context. For example, Wu (2001) and Wu and Plantinga (2003) consider city formation when people have a taste for proximity to exogenously located open space, such as a park. Their analyses focus on the role of open spaces in city structure, but they do not consider the possibility for the available amount of open spaces to be modified by the choice of location of people, concealing the fact that people impose external costs on each other. Strange (1992) and Marshall (2004) consider the question of open space in city in a model with housing externalities but they do not model a land market. Walsh (2007) proposes a Tiebout model in which people have preferences over the characteristics of neighborhood landscape (the amount of open spaces in particular) but the Tiebout framework does not allow to analyse the micro-structure of urban development that we want to develop here. Justifying theoretically the fight against urban sprawl, Brueckner (2000) explains that the social value of open space is not taken into account by households when they make their choice of location, leading to an excessive extension of city. Hence, he recommends to limit urban sprawl in order to preserve open spaces that are located at the outskirts of cities, such as agricultural plains or forests. Several others papers focus instead on the role of intra-urban open spaces : in a theoretical framework, Turner (2005) analyzes the equilibrium and optimum city structure when households value local open space, and he shows that the optimal city is less compact than at the private equilibrium. According to him, public policies such as urban growth boundaries are not fitted when households value local open space. Cavailhès et al. (2004) also demonstrate that the sprawled pattern of cities and the existence of a periurban area is the consequence of households' preferences for natural amenities near their place of residence and thus is not necessarily an inefficient pattern of development. In a recent study, Caruso et al. (2015) analyse urban forms in a 2D microeconomic model where households value open space close to their location; they show that high preferences for green spaces increase both leapfrog development and the size of the leaps. Thus, to the extent of our knowledge, the question of the preservation of different types of open space in a sole model, and the analysis of land sparing and land sharing in a urban context is not yet covered in the economic literature.

This paper expands the literature by developing a theoretical urban model which takes into account explicitly the existence of two different types of open spaces for which preservation strategies may be conflicting. Firstly, we study the impact of households' preferences for proximity open spaces - such as parks, greenways, public gardens and natural or agricultural land directly visible from their place of residence - on the equilibrium city structure. In a second step, we introduce the role of large open spaces outside of cities such as forests, meadows and agricultural land, which are valued by a social planner for their ecological interest. We study the optimal city structure when the social planner takes into account biodiversity conservation. We applied here our model to biodiversity conservation, making the hypothesis that households value only local open space for recreational amenities while the social planner value open space for environmental amenities. Howewer, the model could be extanted to every context where the private incentive to develop is different than the social planner incentive. Thus, this model could work for every amenity related to space occupation. For example, social planner could also value open space for recreational amentities, but in a different way than household : it could be the case in forests or nature reserves.

The remainder of the paper is organized as follows. Section 2 presents the general structure of the model. Section 3 provides an analysis of the equilibrium city structure. Section 4 develops the welfare analysis. Finally, section 5 concludes.

2 The model

2.1 Residential behavior

Consider a city lying on a uni-dimensional space $X = [0, x_L[$, where x_L is the maximum boundary of the city (either for physical or administrative reasons). The city is monocentric: all the firms locate at the central business district (CBD), located at 0 and which size is neglected [1]. At each location $x \in X$, the quantity of available land is equal to one. Our objective is to describe the pattern of the residential area in this city, that we assume closed: the number of households is fixed and the equilibrium utility level varies endogenously. The assumption of a closed city is relevant in order to study the possible allocations of different land-uses within the city. Our model is based on the basic Alonso-Muth-Mills model. All households are assumed to be homogeneous, meaning that each household's income level and utility function are identical. Households divide their entire

[1] Several models where firms' location is endogenous exist in the economic literature (Fujita, 1989), the location of firms may matter when they affect environmental damage, as for instance air pollution (see (Regnier and Legras, 2017)). However we assume here that the location of firms is not relevant since their location choice is not influenced by open space.

income between the consumption of a composite good, a residential good, and commuting costs, proportional to the distance to the CBD. The lot size of each house is assumed to be exogenously fixed.

The model, like all models, simplifies a more complex reality and has limitations. It is, of course, self-evident that real-world cities are characterized by heterogeneity in incomes and housing, but the basic insights of the model remain even in models that allow for these kinds of heterogeneity (Anas, 1990). Another simplification is that the model is static; it does not account for the long-lived nature of the housing stock, nor the process of urban change and phenomena such as filtering or gentrification. Instead, the model is best viewed as providing insights into the longer-run determinants of the urban equilibrium.

The model's simplicity helps focus on fundamental forces, likely to matter for all large cities. Indeed, as Brueckner (1987) argues, this simple yet powerful model explains the main observed regularities of the urban structure.

In order to study the provision of open space in the city, we propose an extension of the model to take into account the fact that a residential area is also characterized by the presence of local open space, which is inversely proportional to the level of development at each location. We consider that households have preferences for local open space, available directly at their place of residence. Natural amenities from local open space are considered as spatial attributes of housing, which affect the households' utility function but not their budget constraint.

Following Fujita (1989), households' maximization program is given by :

$$\begin{cases} \max\limits_{m,x} & u(m,q,d(x)) \\ s.t. & p(x)q + m + t(x) \leq w \end{cases}$$

with:

$u(\cdot)$: the utility function

x : the distance from the CBD

m : the amount of the composite good, which price is the numéraire

q : the lot size of the house, assumed to be fixed

$d(x)$: the level of urban development at location x, with $0 \leq d(x) \leq 1$

$p(x)$: the rental price of a house at distance x

$t(x)$: commuting costs for a household located at distance x

w : the income

The amount of local open space is inversely correlated with the level of urban development at location x and thus is a decreasing function of $d(x)$.

At equilibrium, the utility level is given by u^*, and is equal among all households no matter their residential location as they are identical and mobile without cost. The household's demand function for the composite good $m^*(q, d(x), u^*)$ is obtained by solving:

$$u(m, q, d(x)) = u^* \tag{1}$$

We can now derive the residential bid-rent function of households $p(x)$, which is a generalized form of the bid rent function originally defined by (Alonso, 1964) and indicates the maximum amount a household is willing to pay at location x while receiving the utility level u^* :

$$p(x) = \frac{w - t(x) - m^*(q, d(x), u^*)}{q} \tag{2}$$

The residential bid-rent is an implicit function of the income, the commuting cost, the lot size, and the level of urban development: $p(x) \equiv p(w, t(x), q, d(x), u^*)$, with $\frac{\partial p}{\partial w} > 0$, $\frac{\partial p}{\partial t(x)} < 0$, $\frac{\partial p}{\partial q} > 0$, and $\frac{\partial p}{\partial d(x)} < 0$. When prices vary accordingly across the city, households' utility is identical across locations and households have no incentive to move.

The bid-rent function reveals the difference between our model and the standard monocentric city model. In the standard model, natural amenities are assumed to be distributed uniformly across the landscape : residential rents always fall with the distance from the CBD, compensating residents for their increasing cost of commuting. However, with spatial variations in open space amenities, the spatial pattern of the rent is more complex. A household may be willing to sacrifice proximity to the workplace for open space, with the result that the willingness to pay for housing may no longer be a monotonically decreasing function of the distance to the CBD. This result has been discussed in detail in the seminal paper of Richardson (1977) and in Wu et al. (2004); we will discuss it in section 3.2.

Let's consider a log-linear utility function of the following form :

$$U(m,q,d(x)) = \alpha \ln(m) + \beta \ln(q) + \gamma \ln\left(\frac{1}{d(x)}\right) \tag{3}$$

where α, β and γ are positive constant, and linear transport costs $t(x) = tx$. Households' bid-rent function is given by:

$$p(x) = \frac{1}{q}\left[w - tx - e^{\frac{u^*}{\alpha}} q^{\frac{-\beta}{\alpha}} \left(\frac{1}{d(x)}\right)^{\frac{-\gamma}{\alpha}}\right] \tag{4}$$

It is decreasing with transport cost t according to: $\frac{\partial p}{\partial t} = \frac{-x}{q}$, and decreasing with the level of development at each x according to: $\frac{\partial p}{\partial d(x)} = \frac{-e^{\frac{u^*}{\alpha}} q^{\frac{-\beta}{\alpha}} \left(\frac{1}{d(x)}\right)^{\frac{-\gamma}{\alpha}} \gamma}{\alpha d(x) q}$. The impact of the distance from the CBD will be analysed once the development level function is established in the developer's program.

2.2 Development decision

On the supply side, housing is produced with land, labor and materials under constant returns to scale. The house size, q, is fixed, and outside-developers choose the level of development $d(x)$ at each location, which is equivalent to the number of dwellings per acre. We assume that at each location, only one developer is the landowner of the parcel and takes the development decision. Moreover, the cost of development $c(d(x))$ is assumed a linear function of the development density $d(x)$ such that $c(d(x)) = Cd(x)$ where C is a positive constant, meaning that the cost increases at the same rate as $d(x)$.

At each location, the developer chooses the development density to maximize profit :

$$\max_{d(x)} \pi(d,x) = p(w, t(x), q, d(x), u^*)d(x) - Cd(x)$$

The equilibrium development density is the solution of the first order condition given by:

$$\frac{\partial p(\cdot)}{\partial d(x)} d(x) + p(\cdot) = C \tag{5}$$

From the second order condition, we obtain that $\frac{\partial^2 \pi}{\partial d(x)^2} < 0$ as long as the rent is a decreasing and concave or decreasing and linear function of $d(x)$. Then the profit function reaches its maximum when $d(x)$ is the solution of the differential equation (5).

Using the implicit function theorem, we derive the variation of the level of development along the city :

$$\frac{\partial d(x)}{\partial x} = \frac{\frac{\partial^2 \pi}{\partial d(x) \partial x}}{\frac{-\partial^2 \pi}{\partial x^2}} \tag{6}$$

Using the second order condition, we know that the denominator of the above equation is always positive. Then the sign of (6) depends on the sign of the numerator :

$$\frac{\partial^2 \pi}{\partial d(x) \partial x} = \frac{\partial^2 p}{\partial d(x) \partial x} d(x) + \frac{\partial p}{\partial x} \tag{7}$$

The first part of the right hand side of (7) corresponds to a cross effect. It is interpreted as how the variation of the bid-price with respect to $d(x)$ changes along the city gradient. The sign of this term is a priori unknown and depends on the functional from specified in the household's program. The second term of (7) corresponds to the variation of the housing price along the city that does not depend on the level of development $d(x)$, it passes trough the commuting cost and it is always negative. As a consequence, the sign of the total variation of the development level with respect to the distance to the city center depends on the sign of the cross-effect. If the cross-effect is nul or negative, the level of development always falls along the city gradient. If the cross-effect is positive, the level of development might increase as we move away from the city center.

The first order condition is a second order differential equation of the variable $d(x)$, its solution is given by :

$$d^*(x) = \frac{p(w, t(x), q, d^*(x)) - C}{K} \tag{8}$$

where K is an unknown constant, as we have not defined any specific functional form.

The development density is a function of the residential rent and, through it, the level of amenities at each location. Indeed, households have preferences for local open space, as indicated by (4), meaning that the development density is a disamenity for households and an increase in development density reduces households' willingness to pay for housing. In that case, the developer chooses the number of houses built by balancing households' taste for local open space and her own interest for high density. Thus, when households value local open space, the level of development at equilibrium is not maximal in each location x of the city. For some x, only a part of the parcel is converted to residential use. This result is fully developed in the analysis of the equilibrium land allocation in Section 3.

Using the log-linear utility function described in (3), the equilibrium development level writes as follows:

$$d^*(x) = \frac{\alpha}{\alpha + \gamma} \left[(w - tx - Cq) e^{\frac{-u^*}{\alpha}} q^{\frac{\beta}{\alpha}} \right]^{\frac{\alpha}{\gamma}} \tag{9}$$

from which we can derive:

$$\frac{\partial d^*(x)}{\partial x} = \frac{\alpha^2 t}{-\gamma(w - tx - Cq)(\alpha + \gamma)} \left[(w - tx - Cq) e^{\frac{-u^*}{\alpha}} q^{\frac{\beta}{\alpha}} \right]^{\frac{\alpha}{\gamma}} \tag{10}$$

Under this specific functional form, as long as $w - tx - Cq > 0$, the level of development always decreases with distance to the city center at equilibrium. The negative slope of the development density results from a trade-off between the commuting cost and the level of open space in each x. This trade-off reflects households' preferences for open space, and it affects the equilibrium level of development chosen by the developer. Thus, this equilibrium development density is also the result of the developer's trade-off between maximizing the number of houses built and gaining the maximum possible rent from each house.

It is also interesting to note that by choosing the level of urban development, developers are able to influence the value of the residential rent: they have market power. It is not surprising as the model can be seen as a one of horizontal differentiation, where developers can differentiate land depending on the amount of open space they leave in the parcel. Consequently, they make a positive mark-up (see equation (5)) leading to a market imperfection. However, this market imperfection does not necessarily lead to a deacrease in welfare, as the developers' markup is compensated by the gain in households' utility due to product differentiation. Comparing the development function derived above and the one obtained without market power (by setting $C = p(.)$), we obtain that land development without market power is twice than with market power.

3 The urban-periurban-rural equilibrium

Here we investigate in more details the possible configurations of the city at equilibrium, especially we demonstrate that a urban-periurban-rural equilibrium can arise.

At equilibrium, housing prices are bid up until no household has any incentive to move. This condition is satisfied when housing prices are represented by (4) since the household's bid-rent function is their maximum willingness to pay for housing. At each location x, landowners choose the use that maximizes the return of their plot of land. Therefore, land is developed if the return in residential use exceeds the return of land in its previous state (agricultural or natural), which is supposed to equal zero.

The return per acre in residential use at a particular location x is given by the developers' profit at equilibrium and defined as :

$$r_c(w,t,q,u^*,d^*(x)) = p(w,t,q,u^*,d^*(x))d^*(x) - Cd^*(x) \tag{11}$$

The first closing condition of the model states that residential rent must equal the exogenous agricultural rent at the city boundary x_m :

$$r_c(w,t,q,u^*,d^*(x_m)) = 0 \tag{12}$$

The second closing condition requires that all households live in the city :

$$\int_0^{x_m} n(x)dx = N \tag{13}$$

Where $n(x)$ is the equilibrium number of households at any location and equals the equilibrium development density divided by floor space per household : $n(x) = d^*(x)/q$, and N is the total number of households in the city and is fixed exogenously. Moreover, the following condition must be satisfied : $Nq \leq x_L$, where x_L is the maximum boundary of the city (either for physical or administrative reasons). This conditions means that the total floor space occupied by city's residents cannot exceed the maximum boundary of the city.

The total value of land in the city, R, is given by the total value of land in residential use :

$$R = \int_0^{x_m} r_c(w,t,q,u^*,d^*(x))dx \tag{14}$$

3.1 Equilibrium level of development

The compact built-up area At equilibrium the level of urban development in each x is given by (9), reflecting the trade off between transport cost and the amount of open space in each x. It is possible that for some x, the transport cost is so low that the level of development x reaches its maximum level, so equal to 1 :

$$d^*(x_u) = \frac{\alpha}{\alpha+\gamma}\left[(w - tx_u - Cq)e^{\frac{-u^*}{\alpha}}q^{\frac{\beta}{\alpha}}\right]^{\frac{\alpha}{\gamma}} = 1 \tag{15}$$

$$\Leftrightarrow x_u = \frac{\alpha(w - Cq) - b(\gamma+\alpha)}{\alpha t} \tag{16}$$

Where $b = q^{\frac{-\beta}{\alpha}}e^{\frac{u^*}{\alpha}}$. Thus, for every $x \leq x_u$, there is no open space left, and the level of development $d^*(x)$ is equal to 1. The parcel x_u delimitates the frontier of the compact built-up area in the city. We need to check under which condition this frontier x_u is greater than zero to ensure that there exists a compact built-up area at equilibrium :

$$x_u > 0 \tag{17}$$

$$\Leftrightarrow \gamma < \frac{\alpha(w - Cq - 1)}{b} \tag{18}$$

If γ is low enough, meaning that households have moderate preferences for open space available at their place of residence, there exists a compact built-up area at equilibrium. Otherwise, if γ is too high, households have very strong preference for open space and there is no possibility for a compact built-up area to exist at equilibrium.

The periurban area The periurban area is where the level of development varies between 0 and 1. Recall that $\frac{\partial d(x)}{\partial x} = \frac{\alpha^2 t}{-\gamma(w-tx-Cq)(\alpha+\gamma)}\left[(w - tx - Cq)e^{\frac{-u^*}{\alpha}}q^{\frac{\beta}{\alpha}}\right]^{\frac{\alpha}{\gamma}}$, meaning that close to x_u, the level of development is high and close to 1, and it decreases along the city until it equals zero at the city's limit x_m.

The rural area The city's limit x_m is reached when $R_c(x) = 0$: when developers have no interest to develop houses because the residential return is no longer higher than the agricultural return. the level of development $d^*(x)$ is equal to zero :

$$R_c(x_m) = 0 \tag{19}$$

$$\Leftrightarrow x_m = \frac{w - Cq}{t} \tag{20}$$

Thus, for every $x \geq x_m$, the level of urban development is null.

The level of development along the city is depicted in Figure 1.

3.2 Total value of land at equilibrium

In the general framework, return per acre in residential use varies with distance to the city center according to :

$$\frac{\partial r_c}{x} = \underbrace{\left(\frac{\partial p}{\partial t}\frac{\partial t}{\partial x} + \frac{\partial p}{\partial d^*}\frac{\partial d^*}{\partial x}\right) d^*(x)}_{\text{price effect}} + \underbrace{p(\cdot)\frac{\partial d^*}{\partial x} - C\frac{\partial d^*}{\partial x}}_{\text{size effect}} \tag{21}$$

We can decompose this expression as follows.

The first part of the right-hand side is a price effect. The price paid by households varies with distance according to variations of commuting cost and urban development. They both act as negative forces on households' bid-rent function. However, commuting cost increases with distance, while the level of urban development can increase or decrease, as demonstrated above. If the level of development decreases with distance, the price effect is always negative. However, if the level of development increases with distance, the total effect on the price paid by households is unknown, and it depends of the rate of variation of commuting cost relative to that of urban development. This result is similar to the one demonstrated in the seminal paper by Richardson (1977), in which the variation of residential rent in the presence of externalities are analysed.

The second part of the right-hand side corresponds to a size effect: the return in residential rent depends on the share of land that developers choose to built $(d(x))$ which is decreasing with distance. Thus, the revenue of developers decreases with distance, but so do their costs. In a classic model of production the size effect is nul as the marginal revenue equals the marginal costs at equilibrium. However in our model the size effect is negative, because of the market power held by developers (see (5)).

The total variation of the residential return with respect to the distance to the city center is thus a priori unknown with no specific functional forms. Let's analyse it with a linear utility function. At equilibrium, the return of land in residential use is given by :

$$r_c(x) = \left[\frac{w - tx}{q} - e^{\frac{u^*}{\alpha}} q^{\frac{\beta}{\alpha}} \left(\frac{1}{d(x)} \right)^{\frac{-\gamma}{\alpha}} \right] d^*(x) - C d^*(x)$$

In the compact built-up area, the level of development $d^*(x)$ is equal to one. In that case, their is no difference with the classic urban economics model with no open space and fixed lot size, and the rent is linearly decreasing with the distance to the CBD according to the variation of transport cost :

$$\frac{\partial r_u^*(x)}{\partial x} = \frac{-t}{q} \tag{22}$$

In the periurban area, the level of development $d^*(x)$ varies between zero and one and it affects the equilibrium residential bid-rent. Here, we have :

$$\frac{\partial r_p^*(x)}{\partial x} = \frac{kt(w - tx - Cq - g)(\alpha + \gamma)}{-\gamma(w - tx - Cq)q} \tag{23}$$

Where, $k = \left(\frac{\alpha(w-tx-Cq)e^{\frac{-u^*}{\alpha}} q^{\frac{\beta}{\alpha}}}{\gamma+\alpha} \right)^{\alpha/\gamma}$ and $g = e^{\frac{-u^*}{\alpha}} q^{\frac{\beta}{\alpha}} \left(\frac{1}{k}\right)^{\frac{-\gamma}{\alpha}}$.

Thus, if $w - tx - Cq > g$, the residential rent decreases with distance to the CBD, but at a slower rate than in the compact built-up area. This phenomenon is explained by households' preference for open space : when households locate far away from the center, they pay high transport cost, but they trade-off with the amount of open space enjoyed. Thus, the rent is decreasing slowly because open spaces create a positive force on the equilibrium rent. When households have preference for proximity open space, the city extends more than in the classic Muth-Mills model of urban economics. In our case however the slope of the rent remains negative, because the rate of variation in the amount of open space is not greater than the rate of variation in the transport cost as long as the previous conditions is satisfied.

Finally, in the rural area, for $x \geq x_m$, the equilibrium rent is equal to the land rent in its agricultural state, here equal to zero for simplicity. The variation of the residential return along the city is depicted in Figure 2, where x_c represents the city's limit in the classic monocentric urban economics model.

4 Effects on biodiversity and welfare analysis

Until now, we have considered only the presence of intra-urban open space that are valued by households when they make their choice of location. However, the social planner cares also for big natural open space at the outskirts of city (Brueckner, 2000). Indeed, he values both type of open spaces because they offer provision and regulation ecosystem services, such as habitats for natural species. We define the level of biodiversity as follow :

$$B(d(x), x_m) = \int_0^{x_m} w_1[1 - d(x)]dx + w_2(L - x_m) \tag{24}$$

The first term of the right hand side of the equation represents the contribution of intra-urban open space, while the second term represents the contribution of the big open space at city's outskirts; w_1 and w_2 are the weights associated with each type of open space regarding biodiversity conservation and are assumed to be positive. These weights depend on the social planner's priorities on biodiversity conservation. Indeed, the presence of some species may be favored with small patches of intra-urban open space, while other species may prefer large open space.

Drawing upon the methodology of Fujita (1989), the problem of the social planner is to choose the level of urban development $d(x)$ at each x and the urban fringe distance x_m

so as to maximize the sum of the total surplus and the biodiversity level while achieving the target utility u for N households [2]:

$$\begin{cases} \max\limits_{d(x),x_m} \quad \int_0^{x_m} \left[w - tx - e^{\frac{u^*}{\alpha}} q^{\frac{-\beta}{\alpha}} \left(\frac{1}{d(x)} \right)^{\frac{-\gamma}{\alpha}} \right] \frac{d(x)}{q} dx - \int_0^{x_m} Cd(x)dx + B(d(x), x_m) \\ \\ s.t. \qquad \int_0^{x_m} \frac{d(x)}{q} = N \end{cases}$$

The Lagrangian function associated to the above maximization program is :

$$\mathcal{L}(d(x), x, \lambda) = \int_0^{x_m} \left[e^{\frac{u^*}{\alpha}} q^{\frac{-\beta}{\alpha}} \left(\frac{1}{d(x)} \right)^{\frac{-\gamma}{\alpha}} \right] \frac{d(x)}{q} dx - \int_0^{x_m} Cd(x)dx \\ + w_1 \int_0^{x_m} (1 - d(x))dx + w_2(L - x_m) + \lambda(N - \int_0^{x_m} \frac{d(x)}{q}) \quad (25)$$

The optimal conditions of this problem are given by :

$$d(x) = \begin{cases} 1 & \text{for } x \leq x_u \\ d^*(x) = \frac{\alpha}{\alpha+\gamma} \left[(w - tx - Cq - w_1 q + \lambda) e^{\frac{-u^*}{\alpha}} q^{\frac{\beta}{\alpha}} \right]^{\frac{\alpha}{\gamma}} & \text{for} \quad x_u \leq x \leq x_m \\ 0 & \text{for } x > x_m \end{cases} \quad (26)$$

$$\int_0^{x_m} \frac{d(x)}{q} = N \quad (27)$$

We now compare the optimal conditions with the equilibrium results. We see that the level of urban development inside the peri-urban area is different at the optimum than at equilibrium. Indeed, the optimal level of urban development decreases with w_1, which is

[2]In spaceless economics, it is common to maximize a Benthamite social welfare function, which is the sum of utilities of individual households. However, as explained in Fujita (1989) (p.63) this is not the most convenient approach to land use problems, because the maximization of a Benthamite welfare function results in the assignment of different utility levels to identical households depending on their locations. Since competitive markets treat all equals equally, a more convenient formulation of optimization problems for land use theory is the so-called Herbert-Stevens model. In this model, the objective is to maximize the surplus subject to a set of specified target utility levels for all household types.

the marginal contribution of the intra-urban open spaces to the preservation of biodiversity, and increases with λ which is linked to the population constraint and represents the marginal net cost of a household in the city.
Moreover we see that $\frac{\partial d(x)}{\partial x} = \frac{-t}{2q\gamma}$ for $x_u \leq x \leq x_m$, meaning that at the optimum the urban development decreases at a lower rate with the distance to the city center compared to equilibrium.

Solving the boundary condition, we obtain the optimal city's limit, that we call x_{m_o}:

$$x_{m_o} = \frac{1}{\alpha t}\left[-Ab(\gamma^{\frac{\alpha}{\gamma+\alpha}} + \alpha\gamma^{\frac{\gamma}{\gamma+\alpha}}) + \alpha(w - Cq - w_1 q + \lambda)\right] \tag{28}$$

where $A = \left((w_2 - w_1)\alpha e^{\frac{-u^*}{\alpha}} q^{\frac{\beta+\alpha}{\alpha}}\right)^{\frac{\gamma}{\gamma+\alpha}}$ and b defined as previously.

Using the first two equations of system (26), we also obtain x_{u_o}, the optimal limit of the compact built-up area :

$$x_{u_o} = \frac{1}{\alpha t}\left[-b(\gamma + \alpha) + \alpha(w - Cq - w_1 q + \lambda)\right] \tag{29}$$

It is easy to compare the limit of the urban core at equilibrium x_u, with the one at optimum x_{u_o}. We see that two new terms appears in x_{u_o} : one related to the biodiversity provided by proximity open space, and the other related with the population constraint. More precisely, the compact built-up area becomes smaller when the marginal gain of biodiversity provided by local intra-urban open spaces increases, and becomes larger when the net cost of a households in a city increases. The size of the compact built-up area is not directly related to the marginal gain of biodiversity provided by big open spaces.

Comparing the optimal and the equilibrium limits of the whole city is more complicated, but we can see that at optimum, the city's limit x_{m_o} depends on w_1, w_2 and λ as follows :

$$\frac{\partial x_{m_o}}{\partial w_1} = \frac{-Ab\gamma^{\frac{2\alpha+\gamma}{(\alpha+\gamma)^2}} + \alpha\gamma^{\frac{\alpha}{(\alpha+\gamma)^2}})}{\alpha t(w_1 - w_2)} - \frac{q}{t} \tag{30}$$

Here, we see two effects : the first term of the right hand side of above equation is positve as long as $w_2 > w_1$, and means that city size increases if the contribution of intra-urban open space increases, as more intra-urban open spaces are needed at the optimum. The second term of the right hand side is negative and corresponds to a mechanical effect : if w_1 increases, the frontier of the pure urban core x_{u_o} is lower ($\frac{\partial x_{u_o}}{\partial w_1} < 0$), and its has thus a negative effect on x_{m_o}.

$$\frac{\partial x_{m_o}}{\partial w_2} = \frac{Ab\gamma^{\frac{2\alpha+\gamma}{(\alpha+\gamma)^2}} + \alpha\gamma^{\frac{\alpha}{(\alpha+\gamma)^2}})}{\alpha t(w_1 - w_2)} \tag{31}$$

As long as $w_2 > w_1$, the variation of x_{m_o} with respect to w_2 is always negative. The higher the marginal contribution of big outskirts open space to biodiversity, the smaller the city.

$$\frac{\partial x_{m_o}}{\partial \lambda} = \frac{1}{t} \tag{32}$$

The variation of x_{m_o} with respect to λ is always positive. The higher the net cost of a household in the city, the larger the city. The net cost of a household in the city, λ, can intuitively be interpreted as the strength of the population constraint. Thus, the stronger the population constraint, the larger the city to accommodate all the households.

When x_{uo} equals zero, the city structure is complete land sparing. We have :

$$\begin{aligned}
&x_{uo} = 0 \\
\Leftrightarrow\ &\frac{w_1}{w_2} \geq \frac{\alpha(w - Cq - \alpha q) - b(\alpha+\gamma)}{w_2\alpha q} \\
\Leftrightarrow\ &\frac{w_1}{w_2} \geq (\frac{w_1}{w_2})^h
\end{aligned}$$

When x_{uo} equals to x_{um} the city structure is complete land sparing. We have :

$$\begin{aligned}
&x_{uo} = x_{um} \\
\Leftrightarrow\ &\frac{w_1}{w_2} \leq 1 - \frac{\psi}{w_2}\left(\gamma^{\frac{2\alpha+\gamma}{(\alpha+\gamma)}} + \alpha\gamma^{\frac{\alpha}{(\alpha+\gamma)}}\right)
\end{aligned}$$

$$\Leftrightarrow \frac{w_1}{w_2} \leq (\frac{w_1}{w_2})^l$$

Where $\psi = \frac{1}{\alpha(\alpha+\gamma}\left(q^{\frac{\alpha-\beta}{\alpha}} e^{\frac{u^*}{\alpha}} \left(\frac{\gamma+\alpha}{\gamma^{\frac{\alpha}{\gamma+\alpha}}+\alpha\gamma^{\frac{-\gamma}{\gamma+\alpha}}}\right)^{\frac{\alpha}{\gamma}}\right)$

Proposition 1 1. *If the marginal gain of biodiversity provided by intra-urban open space, w_1, is large relatively to the marginal gain of biodiversity provided by big outskirts open space, w_2, such that $\frac{w_1}{w_2}$ is higher than $(\frac{w_1}{w_2})^h$, the optimal city structure is complete land sharing.*

2. *If the marginal gain of biodiversity provided by intra-urban open space, w_1, is low relatively to the marginal gain of biodiversity provided by big outskirts open space, w_2, such that $\frac{w_1}{w_2}$ is lower than $(\frac{w_1}{w_2})^l$, the optimal city structure is complete land sparing.*

3. *If $(\frac{w_1}{w_2})^h \geq \frac{w_1}{w_2} \geq (\frac{w_1}{w_2})^l$, the optimal city structure is a mixed between complete land sharing and complete land sparing. The closer $\frac{w_1}{w_2}$ is to $(\frac{w_1}{w_2})^h$, the more the land is spared, and the closer $\frac{w_1}{w_2}$ is to $(\frac{w_1}{w_2})^l$, the more the land is shared.*

Proposition 1 shows that the optimal city structure can take several forms. The equilibrium city structure can either be too compact, or too sprawled, compared to the optimum, depending on the relative marginal contribution of the different types of open spaces on biodiversity conservation.

A question arising now is to know how the boundaries of the ratio of the marginal contribution of open space, namely $(\frac{w_1}{w_2})^h$ and $(\frac{w_1}{w_2})^l$ vary with the parameters of our model, in order to understand the impact of these parameters on the resulting optimal city structure. Parameters of interest are the revenue w, the lot size q and the preference for open space γ.

Here it is important to note that the expression of the optimal city's limit (28) implies that w_2 must always be larger than w_1 in order to obtain an explicit solution. This come from the fact that our biodiversity function is a linear function of the provision of each type of open space. Thus, this result means that in our model, one unit of open space at city's outskirt provide more biodiversity than one unit of open space inside the city. This hypothesis is true in cities where extra-urban open spaces are mostly in natural states, for example natural forests or natural grasslands. However, these hypothesis restricts the study of cities where extra urban open space are intensive agricultural land. The contribution of this type of land to biodiversity may be very low, and it is possible that the contribution of intra urban open space is greater in reality, especially if intra-urban open space are grassy plots and trees. In order to study fully this scenario, we must write the biodiversity function under a more complex non-linear form. However, it would not be possible in that case to provide full analytical results and interpretation. Thus, we accept here to maintain the hypothesis that $w_2 > w_1$,

Knowing that $w_2 > w_1$, and in order to ease the analysis, we suppose now that w_2 is maximum and equals one. In that way, we can rewrite the boundaries with regard to w_1 only, and we have :

$$\frac{\partial w_1^h}{\partial w} = \frac{1}{q} > 0 \tag{33}$$

$$\frac{\partial w_1^l}{\partial w} = 0 \tag{34}$$

An increase in the revenue of households has for consequence to rise the boundary above which the city in complete land sharing, meaning that this configuration will be less likely an optimal configuration. Indeed, household would be ready to accept less open spaces near their house while deriving the same utility because they have a greater revenue. The variation of the revenue has no effect on the boundary under with the optimal city is complete land sparing.

$$\frac{\partial w_1^h}{\partial q} = \frac{b(\gamma\beta + \alpha\beta + \alpha\gamma + \alpha^2) - \alpha^2(w+\lambda)}{\alpha^2 q^2} \tag{35}$$

$$\frac{\partial w_1^l}{\partial q} = q^{-\frac{\beta+2\,\alpha}{\alpha}}\,(\beta+\alpha)\,e^{\frac{u}{\alpha}}\left(\frac{\gamma+\alpha}{\gamma^{\frac{\alpha}{\gamma+\alpha}} + \gamma^{-\frac{\gamma}{\gamma+\alpha}}\alpha}\right)^{\frac{\alpha}{\gamma}} \frac{\gamma^{\frac{2\,\gamma+\alpha}{\gamma+\alpha}} + \gamma^{\frac{\gamma}{\gamma+\alpha}}\alpha}{\alpha^2\,(\gamma+\alpha)} > 0 \tag{36}$$

An increase in the lot size has two effects on the boundary under wich the city is complete land sharing : a positive effect, meaning that this configuration will be more likely optimal, because if q is high, mechanically it reduces the amount of intra-urban open spaces and thus it favors the apparition of complete land sharing cities, all else being equal; the increase in lot size also has a negative effect on the boundary under wich the city is complete land sharing, due to the fact that households will benefits from larger houses and thus will be ready to accept less open space, reducing the possibility for complete land sharing configuration to be optimal, all else being equal.

An increase in the lot size increases the boundary under with the optimal city is complete land sparing, meaning that this configuration will be more likely optimal.

$$\frac{\partial w_1^h}{\partial \gamma} = \frac{-e^{\frac{u}{\alpha}} q^{\frac{-\beta}{\alpha}}}{\alpha q} < 0 \tag{37}$$

An increase in the household's preference for proximity open space has for consequence to decrease the boundary above which the city in complete land sharing.

5 Numerical complements

We provide now some numerical complements in order to illustrate the analytical results presented in the previous section. In all the simulations, the parameters used are $q = 1$, $w = 2.51$, $C = 0.2$, and $\lambda = 0.5$. Table 1 presents different scenarios when w_1, w_2 vary.

Our results suggests that public policies must be carefully designed if the objective is to preserve biodiversity. In the case where the marginal contribution of intra-urban open

	Scenario 1	Scenario 2	Scenario 3
w_1	0.895	0.2	0.3
w_2	0.9	0.9	0.5
u	0.36	0.2	0.25
w^l	0.553	0.553	0.153
w^h	0.893	0.893	0.893
x_{uo}	0	1.089	0.924
x_{mo}	1.812	1.089	1.527

Table 1 – Optimal city structure under different scenarios

space is high (scenario 1), for example in cities where these open space are mostly green parks or grassy plots, optimum is obtained through land sharing configuration. In that case, a policy which provides incentives for infill development of vacant locations would be welfare decreasing. Conservation and preservation of urban parks is the best solution to enhance welfare.

However, the recommendation are different when intra-urban open space provide a low marginal gain of biodiversity (scenario 2), for example in cities where these intra-urban open spaces are predominantly brownfields - such as Detroit in the USA : in that case, land sparing is the optimal city structure. Densification of the core city associated to creation of greenbelts by prohibiting development in wide regions of the periphery appears to be the best option to increase welfare.

This result is less strong if the marginal contribution of biodiversity provided by large open space at cities' outskirt is also low (scenario 3). In that case, the optimal city structure is more often a mix between land-sharing and land-sparing. In our analysis, we focus on the benefit of open space directly related to biodiversity preservation, however, we could also consider other types of environmental issues related to the city structure : for example, many studies (Brueckner, 2000; Larson and Yezer, 2015) show that is we consider congestion issues caused by commuting, unpriced congestion results in a city which occupies too much land. This results is also true if we consider air pollution emmited by commuting (Regnier and Legras, 2017). In that case, land sparing configuration may be recommanded, and policies aimed at preserving open space at the edge of the city could act at a second-best policy to deal with congestion or air pollution externalities.

6 Conclusion

We developed a model in which open space can be split into two categories : local intra-urban open space that are directly valued by households as cultural ecosystem services, and large open space at city's outskirts valued by the social planner for environmental reasons. Our aim was to understand how households' preferences affect the equilibrium city structure, we show that when households value local open space, the city is first composed of a pure urban core where all the land is developed, followed by a periurban area where a part of the land is not developed forming intra-urban open spaces. Finally a rural area completes the equilibrium land-use pattern. This result entails that the city extends more when households value local open spaces, which is directly responsible for the loss of large open spaces at city's outskirt.

We then study what is the optimal city structure when a social planner maximizes social welfare taking into account biodiversity conservation and both types of open spaces.

We show that the optimal structure of the city can be either land sharing or land sparing, depending on the relative marginal impact of each type of open spaces on biodiversity conservation. Thus, welfare maximization does not came necessarily with reducing urban sprawl. We need to take into account the complex relation between urban form and nature preservation. Applied here in the context of biodiversity preservation, this result could be extented to every case where a market failure exists where the private incentive to develop is different than the social planner. Thus, it could be applied to all amentities that could occupied the space. Moreover, we could also extend the analysis to negative externalities of open space, for example the existence of brownfields, in that case the w_1 and w_2 of our model would take nul or negative values.

This result is an invitation to undertake adequate research upstream in order to better grasp and foresee the potential perverse effects associated with the promotion of a single form of sustainable city, as is currently the case with the paradigm of the compact city. Our analysis is a first step in the land sharing vs. land sparing debate in an urban context with economic tools. However, several questions still need to be addressed. In particular, it would be particularly interesting to take into account the possibility of "vertical" densification, or to extend the model in a dynamic analysis to better understand the process of urban development and not only the resulting city structure.

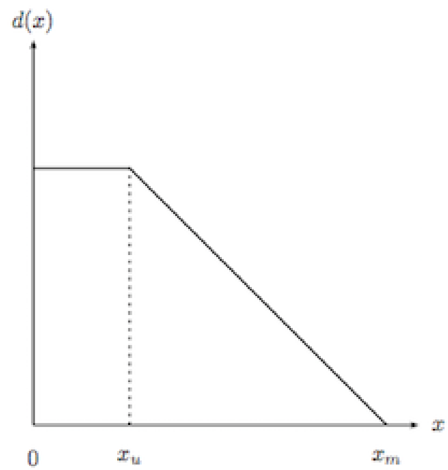

Figure 1 – Variation of urban development along the city

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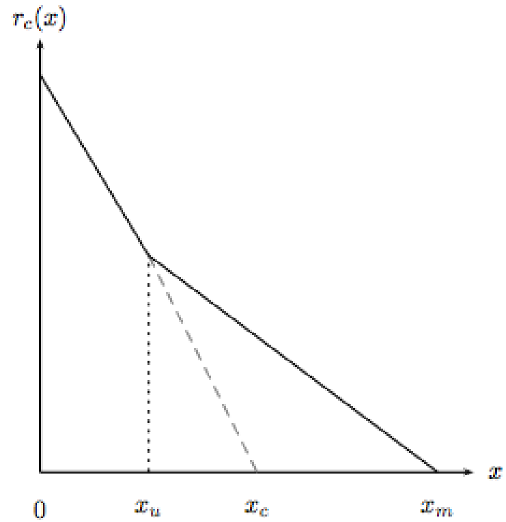

Figure 2 – Residential return gradient

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